§16.1 Chapter Sixteen

Concerning the Logarithms of Sines, Tangents, and Secants.

Logarithms are the attending equidistant numbers of proportional numbers: with the help of which, given any three numbers whatsoever, the fourth proportional is more easily found by addition and subtraction alone. But the whole of trigonometry has been founded on the finding of the fourth proportional number in any of these canons of sines, tangents, or secants. Therefore with these tables we attach the logarithms, as from these the number sought is produced with the least trouble.

But the logarithms of the sines are especially sought conveniently; and from these the logarithms of the tangents and secants can be found.

The different kinds of logarithms for the same numbers can be applied: as shown by me in the first chapter of the *Arithmetica Logarithmica*. The most distinguished man, the Baron of Merchiston who first found these numbers, has explained these which he prepared at that time. And more of the same kind of logarithms have followed by people stepping in the same footsteps; from which Benjamin Ursinus has produced a great and praiseworthy work, these [Logarithms, that follow those of Napier and Kepler] are applied to degrees, minutes, and sixtieths of minutes. I truly was helped with encouragement by this first inventor himself, and I considered the applications of that other kind of logarithms, which have many a use, and which are much easier and more excellent [i.e. those to base 10].

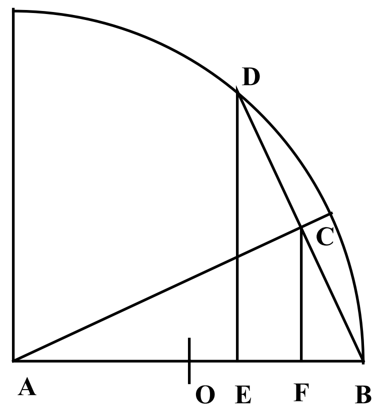
The logarithm is required for the radius of the circle or the total sine, which is set by me as 10,00000,00000,0000. The logarithm agreeing to this radius is set at 100000,00000, of which the characteristic by Chapter 4 of the *Arithmetica Logarithmica* is 10 for the number of places [the logarithm in modern notation is thus 10. 000000000]. The characteristic of the remaining sines truly is 9; we will arrive at this from the sine of 5:44', or 5.73 , from which place as far as to 0:34, or 057 the characteristic is 8. Then truly the characteristic is 7 as far as 0:3 or 0.05. And finally the characteristic is 6 at the beginning of the table. Because with the decreasing sines, which have fewer places for any sine you wish the characteristic is to that least value. But the number of places in this table is more than the characteristic, as we would have the sines themselves more accurate, and finally truly five places are added on to the sines, so that the minutes are not seen to be absent from the perfection of these.

The manner by which any number you wish can have its own logarithm considered, is set out in Chapter 14 of the *Arithmetica Logarithmica*: following which(by dividing the quadrant into 72 equal parts) for the sines of the particular fractions of these, their own logarithms have been furnished; and the other numbers are augmented by quinquisection, to become 360, then 1800, finally 9000; and with the individual degrees assigned 100 logarithms. The method of quinquisection with logarithms is the same which has been with tangents and secants, for which the mean differences are corrected by subtraction only.

If we should wish to display logarithms to thousandths of degrees, the sines have been prepared, which (by the division of the quadrant into 144 small equal parts) of these separately are convenient: and the logarithms themselves can be established for these, and of these the number are increased by quinquisection as before, so they arrive at 720, 3600, 18000, 90000.

But as there is a very large inequality in the differences in the logarithms of the sines at the beginning of the quadrant, it will scarcely be possible by this quinquisection method to show the accurate logarithms of the first degrees. It is therefore to correct this defect, and for no other reason, that this following Proposition is used to compensate for sines themselves, by the golden rule of proportionality; so for the logarithms of the sines, it will serve most conveniently for finding [these] by addition and subtraction alone.

If the sines are given, either from the beginning of the quadrant to 45 degrees: or from the same degree as far as the end: the remainder of the sines can be found by the golden rule. For they are the sine of any arc whatsoever and half the radius of the circle, the mean proportionals between the Sine of the arc being halved, and the Sine of the complement of the same half. As are shown in this diagram



[Figure 16-1]

Let DE be the Sine of 56:0' degrees, BC the sine of 28:0', AC the sine of the complement 62:0', AO half of the radius. The chord BD of 56:0'is drawn , and CF perpendicular to the radius. ABC, ACF are similar triangles, and the lines AB, BC : AC, CF proportionals. In like manner AB, AO: DE, CF are in proportion. And therefore the rectangles BC, AC; AO, DE; AB, CF are equal; and the sides of the equal rectangles are in reciprocal proportion: BC to AO, as DE to AC. [Thus: AB/AO = DE/CF; and AB/BC = AC/CF. From which AO.DE = BC.AC, and BC/AO = DE/AC. Hence, $\sin(\theta/2)/(R/2) = \sin(\theta)/(R\cos(\theta/2)$; or, as Briggs would have it: $\sin(\theta/2)/\sin(30) = \sin(\theta)/\sin(\pi/2 - \theta/2)$] .

If the sine is given from the first quadrant as far as 45 degrees, the remainder of all the dines can be found by the proportional rule, as you see here, if we except the sine of 60 degrees.

However if the sines are given from 45 degrees to the end of the quadrant, the remainder from the beginning as far as 45 degrees are found by the same method.

Proportional Sines			
Given	*Given*	*Given*	*Found*
5	30	10	85
10	30	20	80
15	30	30	75
20	30	40	70
Given	*Given*	*Found Before*	*Found*
35	30	70	55
40	30	80	50
25	30	50	65
30	30	60	60
45	30	90	45
Proportional Sines			
Given	*Given*	*Given*	*Found*
45	30	90	45
50	30	80	40
55	30	70	35
60	30	60	30
65	30	50	25
Given	*Given*	*Found Before*	*Found*
70	30	40	20
75	30	30	15
80	30	20	10
85	30	10	5

[Table 16-1]

3. And in this manner the sine itself can be found; moreover, the logarithms of the same can be found with a little more trouble. Indeed the sines of half the quadrant have been found, and the logarithms of these (by Ch. 14 of the *Arithmetica Logarithmica*): the logarithms of the sines of the whole quadrant can be found by addition or subtraction, or by addition alone. Since indeed for general proportion, the logarithms of the extremes of the ratio are equal to the logarithms of the means, and if from four sines in proportion, three together with their logarithms are given, then the fourth sine and its logarithm can surely and easily be found. I have considered that three examples should be added in order that the method of the operation and the truth of the proposition become clearer. In the first, the first logarithm is taken from the sum of the means. In the two remaining examples, initially the arithmetic complement is put in place, and provides in the fourth place the logarithm sought, by addition alone. We suggest you see Ch. 15 of the *Arith. Log*.

Arc	*Arc*	*Proportional sines*	*Logarithms*
8890	88:54	999998476913288	999999933853134
3000	30:00	500000000000000	969897000433602
020	0:12	3490651415221	754290648129673
010	0: 6	1745328365894	1724187648563275
			724187714710141

Arc	*Arc*	*Proportional sines*	*Logarithms*
8891	88:54:36	999998766299764	000000053578956 *C. A.*
3000	30:00:00	500000000000000	969897000433602
018	0:10:48	3141587485876	747914915830814
009	0: 5:24	1570795680827	1719611969843372

Arc	*Arc*	*Proportional sines*	*Logarithms*
4657	46:34:12	726214813210745	013893489666510 *C. A.*
3000	30:00:00	500000000000000	969897000433602
8686	86:51:36	998498672858271	999934749197014
4343	43:25:48	687467850210673	1983725239297126

[Table 16-2]

The logarithms of the sines that follow are for the individual fractions of the quadrant divided into 72 equal parts, which by quinquisection give rise to 360, 1800, 9000 equal parts (with the help of the previous proposition).

Deg.	Min	Log. Sines	Deg.	1/100th
1	15	833875292857723	1	25
2	30	863967956161593	2	50
3	45	881559852775659	3	75
5	0	894029600833018	5	0
-	-	-	-	-
50	0	988425396655351	50	0
-	-	-	-	-
88	45	999989663737472	88	75
90	0	1000000000000000	90	0

[Table 16-3 (abbreviated)]